Sequences

Some students like to focus on a difference method to identify a formula for the nth term of a sequence. However the structure of a problem is often as, if not more, important than a formula. A favourite way to introduce this idea is to look at the number of crosses on the border and inside of a simple square

This square has three crosses on each side

X   X   X

X   X   X

X   X   X

It has one cross inside the square
and eight crosses on the border

This square has 4 crosses on each side

X   X   X   X

X   X   X   X

X   X   X   X

X   X   X   X

How many crosses does it have on:
- The border?
- The inside?

The task is to identify how many crosses on the border and inside a square for a given number of crosses on one side.

A first task should be to identify the number of crosses on the border and inside for 2, 3, 4, 5, … crosses on one side. What do students notice about these numbers? Are the numbers odd or even, is there a pattern with how the number of crosses increase, are there squared numbers present?

Rather than focusing on the way the numbers increase students can be encouraged to try to identify structure of the crosses on the boarder. For example students could see 4 crosses on each side with an overlap on the four corners:
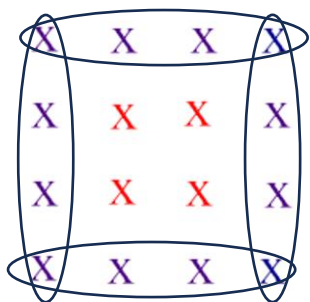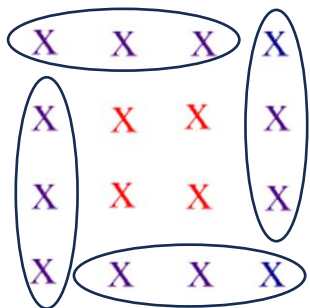

So 4 lots of the number of crosses on each side – 4.

Or students could see the top and bottom of the square and two less from each side:
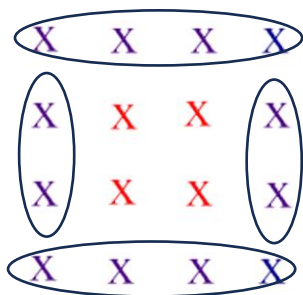
So 2 lots of the number of crosses on each side plus 2 of the (number of crosses on each side – 2).

Another way to see this is the four sets of one less than the number of crosses on each side:

There are other ways to see the structure on the outside and a challenge to students can be to try to identify all the different ways.

Looking at the inside the most common ways to identify the number of crosses is to look at a square with two fewer crosses per edge (number of crosses on each side - 2)^2 or to consider the total number of crosses and then remove the crosses on the border.

This activity can also be used to introduce algebra, using a letter to represent the number of crosses on each side and to show that the different ways to look at the structure of the number of crosses are equivalent.